Probing itinerant ferromagnetism with a ferromagnet/insulator/superconductor junction

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A ferromagnet/insulator/superconductor ballistic junction is used to distinguish the contributions due to exchange splitting and spin-dependent mass renormalization of up- and down-spin bands in itinerant ferromagnets. The study is performed within the Blonder-Tinkham-Klapwijk approach and by solving the corresponding Bogoliubov-de Gennes equations in a way to get the current flux across the junction. The averaged differential conductance is shown to exhibit features that depend on the strength of the mass and the exchange splitting while the knowledge of the transmission critical angle provides a mean to measure the mass asymmetry among up and down carriers.

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: $74.45.+c$, $75.30.-m$, $74.50.+r$, $75.90.+w$

I. INTRODUCTION

The origin of metallic ferromagnetism is nowadays a controversial question. Indeed, it is generally accepted that several distinct mechanisms have to be invoked to describe ferromagnetism in metals in the wide range of manifestations that it exhibits in nature. As relevant examples, we mention the ferromagnetic transition metals Fe, Co, and Ni and their alloys,¹ the weak metallic ferromagnets such as $ZrZn₂$ and $Sc₃In,²$ the colossal magnetoresistance manganites such as $La_{1-x}Sr_xMnO_3$ $La_{1-x}Sr_xMnO_3$,³ and rare-earth hexaborides such as EuB_6 .^{[4](#page-3-3)} Commonly, metallic ferromagnetism has been understood as a competition between single-particle kinetic energy, favoring the paramagnetic state, and the exchange energy originated by the Coulomb interaction, favoring the spinpolarized state[.5](#page-3-4) It is argued that in ferromagnets the gain in exchange energy usually overcomes the cost in kinetic energy due to the Pauli principle that forbids double occupancy of low kinetic-energy states for parallel spins, resulting in an energy split of the majority and minority-spin bands.

An alternative point of view based on the doubleexchange mechanism was proposed to describe ferromagnetism in manganese oxides. 6 In this case metallic ferromagnetism is driven by spin-dependent band renormalization, or equivalently an effective-mass reduction, that occurs upon spin polarization, leading to a gain in kinetic energy. A different mechanism not requiring a multiorbital character may be also invoked as responsible for a kinetically driven ferromagnetism[,7](#page-3-6) namely the bond-charge Coulomb repulsion, which in a tight-binding formulation is related to nearest-neighbor exchange and pair hopping matrix elements of the Coulomb interaction.⁸

How it is possible to extract experimentally the contributions of the exchange splitting and spin-dependent mass renormalization that emerge in the two types of ferromagnet mentioned above? One way is to look at the optical properties;⁹ other useful information could be obtained from angle-resolved photoemission experiments, which can probe band renormalization effects as a function of temperature and magnetization[,10](#page-3-9) as well as from de Haas-van Alphen oscillation measurements.¹¹

In this paper we propose a different tool to analyze exchange splitting and spin-dependent mass contributions to itinerant ferromagnetism. We look at a ferromagnetic/

insulating/s-wave superconducting (F/I/S) junction investigating the role played by the mass splitting and/or the exchange mechanism on conductance curves. The analysis is based on an effective model that can capture the basic aspects of the itinerant ferromagnet upon examination, with two parabolic bands (one for each spin orientation) rigidly shifted relative to each other, including also the possibility for majority and minority carriers to have different masses. Consequences of this spin bandwidth asymmetry have recently been investigated in the context of the proximity effect in a F/S junction in the clean limit as well as in studies on the coexistence of ferromagnetic and superconducting order parameters.¹²

F/S systems have been largely studied in various directions regarding both the symmetry of the superconducting order parameter and the measure of the degree of spin polarization¹³ for the ferromagnetic material looking at the Andreev reflections (AR) (Refs. [14](#page-3-13) and [15](#page-3-14)) or at the tunneling conductance[.16](#page-3-15) Here, the conductance in a F/I/S junction is investigated to probe itinerant ferromagnetism making a clear distinction between the exchangelike case and the mass asymmetry one. While qualitative distinctive signatures can be obtained by inspection of the conductance, more detailed information can be extracted by analyzing the critical angles of the current flux across the junction.^{17,[18](#page-3-17)} In this latter situation we will show that the transmission critical angle allows one to infer the value of the mass-carrier ratio. Otherwise, the AR critical angle cannot be used to discriminate about the character of the ferromagnet since it depends only on its magnetization.

II. MODEL

We use the Blonder, Tinkham, and Klapwijk (BTK) approach¹⁹ to diagonalize the mean-field Hamiltonian for a two-dimensional F/I/S junction on the basis of the Bogoliubov–de Gennes (BdG) equations.²⁰ This approach has been successfully extended in the last years to take into account higher dimensionalities, different symmetries of S order parameter, different Fermi energies for the two sides of the junction, a Stoner ferromagnet in substitution of the normal metal, and a spin–flip interfacial scattering. $21-28$

We confine the analysis to a ballistic planar junction choosing the interface lying in the *y* direction at $x=0$ and built up by two semi-infinite layers connected by an infinitely thin insulating barrier resulting in an interfacial scattering potential of the form $V(\mathbf{r}) = Z\delta(x)$. The region $x < 0$ (from now on the F side) is occupied by a metallic Stoner ferromagnet with possibly different effective masses for each spin orientation, while the region $x > 0$ (from now on S side) is occupied by a conventional singlet *s*-wave superconductor. We describe excitations propagating through the junction by means of the single-particle Hamiltonian H_0^{σ} = $[-\hbar^2 \nabla^2/2m^{\sigma}]$ $-\rho_{\sigma}U - E_F[\Theta(-x) + [-\hbar^2 \nabla^2 / 2m' - E'_F]\Theta(x) + V(\mathbf{r}), \text{ where } \sigma$ $= \uparrow, \downarrow, m^{\sigma}$ is the effective mass for σ oriented electrons in the F side, $\rho_{\uparrow(\downarrow)} = +1(-1)$, *U* is the exchange interaction, E_F is the Fermi energy of the ferromagnet, $\Theta(x)$ is the unit step function, m' and E'_F are the quasiparticles effective mass and the Fermi energy for the superconductor, respectively. The BdG equations read

$$
\begin{pmatrix} H_0^{\sigma} & \Delta \\ \Delta^* & -H_0^{\overline{\sigma}} \end{pmatrix} \begin{pmatrix} u_{\sigma} \\ v_{\overline{\sigma}} \end{pmatrix} = \varepsilon \begin{pmatrix} u_{\sigma} \\ v_{\overline{\sigma}} \end{pmatrix}, \quad \sigma = \uparrow, \downarrow,
$$
 (1)

where $\bar{\sigma} = -\sigma$ and $(u_{\sigma}, v_{\bar{\sigma}}) \equiv \Psi_{\sigma}$ is the energy eigenstate in electron-hole space associated with eigenvalue ε . Considering a rigid superconducting pair potential, i.e., $\Delta(\mathbf{r})$ $=\Delta_0 \Theta(x)$, Eq. ([1](#page-1-0)) admits analytical solution. The Hamiltonian invariance under *y*-directed translations permits to factorize the parallel part of the eigenstate, i.e., $\Psi_{\sigma}(\mathbf{r})$ $=e^{i\mathbf{k}_{\parallel} \cdot \mathbf{r}} \psi_{\sigma}(x)$, hence solving effective one-dimensional equations.

At the interface four scattering processes are possible for an electron injected from the F side with spin σ and momentum $\mathbf{k}_{\sigma}^{+} (k_{\sigma}^{+} = [(2m_{\sigma}/\hbar^{2})(E_{F} + \rho_{\sigma}U + \varepsilon)]^{1/2})$: (a) AR resulting in a hole with momentum $\mathbf{k}_{\bar{\sigma}}^{-}$ $(k_{\bar{\sigma}}^{-} = [(2m_{\bar{\sigma}}/\hbar^2)(E_F + \rho_{\bar{\sigma}}U - \varepsilon)]^{1/2})$ belonging to the opposite spin band and a Cooper pair transmitted in the superconductor; (b) normal reflection; (c) transmission as electronlike quasiparticle with momentum \mathbf{k}'_{σ} $(k'_{\sigma} = [(2m'/\hbar^2)(E'_{F} + \sqrt{\varepsilon^2 - |\Delta|^2})]^{1/2};$ (d) transmission as holelike quasiparticle with momentum \mathbf{k}'_{σ} (k'_{σ} $=[(2m'/\hbar^2)(E'_F-\sqrt{\varepsilon^2-|\Delta|^2})]^{1/2}).$

For standard low-biased F/I/S junctions $E_F, E'_F \ge (\varepsilon, |\Delta|)$ so that one can apply the Andreev approximation and fix the momenta on the Fermi surfaces. The corresponding solutions of BdG equations for the two sides of the junctions are

$$
\psi_{\sigma}^{F}(x) = e^{ik_{\sigma,x}^{F}} \begin{pmatrix} 1 \\ 0 \end{pmatrix} + a_{\sigma} e^{ik_{\sigma,x}^{F}} \begin{pmatrix} 0 \\ 1 \end{pmatrix} + b_{\sigma} e^{-ik_{\sigma,x}^{F}} \begin{pmatrix} 1 \\ 0 \end{pmatrix}
$$
 (2)

$$
\psi_{\sigma}^{S}(x) = c_{\sigma} e^{ik_{\sigma,x}^{F}} \begin{pmatrix} \sqrt{\frac{\varepsilon + \chi}{2\varepsilon}} \\ \sqrt{\frac{\varepsilon - \chi}{2\varepsilon}} \end{pmatrix} + d_{\sigma} e^{-ik_{\sigma,x}^{F}} \begin{pmatrix} \sqrt{\frac{\varepsilon - \chi}{2\varepsilon}} \\ \sqrt{\frac{\varepsilon + \chi}{2\varepsilon}} \end{pmatrix},
$$
 (3)

where $\chi \sqrt{\varepsilon^2 - |\Delta|^2}$ and the probability amplitude coefficients a_{σ} , b_{σ} , c_{σ} , and d_{σ} for the four scattering processes have to be calculated from the boundary conditions

$$
\psi_{\sigma}^{F}(0) = \psi_{\sigma}^{S}(0),\tag{4}
$$

$$
\frac{1}{m_{\sigma}} \frac{d\psi_{\sigma}^{F}}{dx}\bigg|_{x=0} - \frac{1}{m'} \frac{d\psi_{\sigma}^{S}}{dx}\bigg|_{x=0} = \frac{2Z}{\hbar^2} \psi_{\sigma}^{S}(0). \tag{5}
$$

We notice that the mass-asymmetry explicitly influences the spectrum of excitations and the relations of the boundary conditions. The differential conductance at $T=0$ and bias voltage *V*, i.e., the quasiparticle density of states at $\varepsilon = -eV$, is calculated from the ratio between the flux induced across the junction and the incident flux at that bias. The differential conductance spectrum per spin orientation at $T=0$ (relative to conductance of the same junction with $\Delta = 0$) is

$$
G_{\sigma}(\epsilon,\theta) = P_{\sigma} \left(1 + \frac{k_{\sigma,x}^F}{k_{\sigma,x}^F} |a_{\sigma}(\epsilon,\theta)|^2 - |b_{\sigma}(\epsilon,\theta)|^2 \right),
$$
 (6)

where θ is the angle with respect to the normal to the interface, formed by the momentum of electrons propagating from the F side, and $P_{\sigma} = n_{\sigma}/(n_{\uparrow} + n_{\downarrow})$ is the fraction of electrons occupying the σ band of the metallic ferromagnet. The measured conductance takes contributions from a range of angles determined by the experimental conditions. From the conservation of the parallel component of the momentum

$$
k_{\sigma}^{F} \sin \theta = k_{\bar{\sigma}}^{F} \sin \theta_{\bar{\sigma}} = k^{\prime F} \sin \theta_{\sigma}', \tag{7}
$$

where $\theta_{\bar{\sigma}}$ and θ'_{σ} are the AR and the transmission angles, respectively, for electrons propagating with spin σ , we infer the existence of critical angles above which these processes are no more possible resulting, respectively, in virtual AR and normal reflection. The averaged differential conductance for given spin orientation is then defined as^{22}

$$
\langle G_{\sigma}(\varepsilon) \rangle = \int_{-\theta_{C}^{\sigma}}^{\theta_{C}^{\sigma}} d\theta \cos \theta G_{\sigma}(\varepsilon, \theta) / \int_{-\theta_{C}^{\sigma}}^{\theta_{C}^{\sigma}} d\theta \cos \theta, \qquad (8)
$$

where θ_C^{σ} is the transmission critical angle for σ electrons.

The spin bandwidth asymmetry in the F side induced by different effective masses of electrons with opposite spin, directly affects the density of states per spin orientation, and consequently the net polarization. For our two-dimensional metallic ferromagnet we find that the ground-state magnetization $M \equiv P_1 - P_1$ is

$$
M = \frac{(X+1)Y}{X(Y-1) + Y + 1} - \frac{1-X}{X(Y-1) + Y + 1},
$$
(9)

where $X = U/E_F$ and $Y = m_\uparrow / m_\downarrow$. Equation ([9](#page-1-1)) correctly reduces to previous results for a pure Stoner ferromagnet when *Y* \rightarrow 1.²⁷ For 0 \leq *X* $<$ 1, the mass mismatch enhances the net polarization when the up electrons band has a smaller bandwidth than the down electrons one (corresponding to $Y > 1$ and up electrons "heavier" than down electrons) and hinders it the other way around $(Y < 1)$.

III. RESULTS

To check if a F/I/S junction with a mass-asymmetry ferromagnet exhibits distinctive features with respect to a purely exchange-driven ferromagnet, we analyze the transport properties of the junction confining the analysis to the points A and B of the magnetic parameter space reported in

FIG. 1. Isomagnetization curves plotted in terms of the mass mismatch and the renormalized exchange interaction. The lines correspond to isomagnetization curves for *M* = 0, 0.05, $0.1, \ldots, 0.95, 0.99$ from left to right, so that lighter regions are associated with higher magnetization. A and B points represent two different microscopic states corresponding to the same macroscopic magnetization $M=0.75$: A corresponds to a standard Stoner ferromagnet while B represents an almost completely mass-asymmetrydriven ferromagnet.

Fig. [1.](#page-2-0) In this figure we plot isomagnetic curves corresponding to different ways of realizing ferromagnetic configurations having the same magnetization, by varying the relative weights of the mass asymmetry and the exchange contributions. While *Y* fixes the ferromagnetic masses, nothing is said *a priori* on *m'*, so that in principle many cases have to be explored. To purely appreciate the effect of mass asymmetry we neglect Fermi mismatch effects and fix $E_F = E'_F$. Although we have analyzed the differential conductance and the critical angles in all the parameter space, we choose here $M=0.75$, confining the investigation to a pure Stoner case (point A in Fig. [1](#page-2-0)) and to the case of a ferromagnet characterized by a large value of the mass-asymmetry ratio Y (point B). If other points belonging to the same isomagnetization curve are considered, we find that the properties of the corresponding states vary with continuity with respect to the extreme A and B cases.

Assuming $m_1/m' = m'/m_1$ for $Y > 1$, we plot in Fig. [2](#page-2-1) the total averaged conductance $\langle G \rangle = \langle G_1 \rangle + \langle G_1 \rangle$, together with its spin-resolved components, in the metallic $(Z=0)$ as well as in the tunneling limit $(Z=10)$. Although in the tunneling regime (see right panels) the conductance curves show a similar behavior, striking differences emerge within the gap, i.e., for $\epsilon/|\Delta|$ < 1, when the metallic limit is considered. Indeed, in this case the averaged differential conductance is a monotonically increasing function when the F side is a massasymmetry ferromagnet, whereas the same quantity in the pure Stoner regime exhibits an opposite behavior being a decreasing function of $\epsilon/|\Delta|$; of course, the first and second derivatives of the subgap conductance spectra have opposite signs for a standard Stoner ferromagnet and for a massasymmetry driven ferromagnet too. Such signature is also observed at different values of the magnetization. This implies that we may *unambiguously* use a F/I/S junction, in the metallic limit, as an efficient tool to identify the exchange and mass-asymmetry contributions in the F side of the junc-

FIG. 2. (Color online) Averaged differential conductance spectra (full line) in the metallic (left) and in the tunneling (right) limit, calculated at the points A (top) and B (bottom) of Fig. [1.](#page-2-0) Dashed (dotted) lines correspond to up (down) spin component of the conductance.

tion. Nevertheless, this kind of analysis can also be performed looking at the tunneling limit. Indeed, performing a spin-resolved tunneling measurement, 29 in the pure Stoner limit the spin components of the conductance are clearly different whereas when the mass mismatch regime is considered the same quantities are practically equal at low bias.

Now we study from Eq. (7) (7) (7) the angular dependence of the Andreev scattering at the interface with a special attention to the critical values of the AR and the transmission angles. It is well known that an increase in the momentum parallel to the interface leads to suppression of the probability of AR and increase in the probability of normal reflection. We find that the AR angle depends only on the effective value of *M*, i.e., it is constant along isomagnetization curves of Fig. [1](#page-2-0) while the transmission angle is sensible to the relative weights of the magnetization sources. Figure [3](#page-2-2) shows the critical angles for the transmission of up electrons in the superconductor along isomagnetization curves for two different ratios of the

FIG. 3. (Color online) Critical angles for the transmission of spin-up electrons in the superconductor along isomagnetization curves, for $m_{\downarrow} = m'$ (left panel) and $m_{\uparrow}/m' = m'/m_{\downarrow}$ (right panel). The value of the magnetization *M* increases from above and the dotted lines correspond to negative values of the exchange energy *U*.

ferromagnetic masses with respect to the superconducting one. We notice that these curves are plotted as a function of the mass mismatch ratio whereas the exchange energy *U* is varied in such a way to keep the magnetization constant, thus also taking negative values (dashed parts of the curves in Fig. [3](#page-2-2)). In the left panel the case $m_{\downarrow} = m'$ is shown, i.e., down electrons propagate in the superconductor with the same mass while up electrons experience a mass mismatch. The right panel shows the same critical angle for a balanced redistribution of the magnetic masses above and below the superconductor quasiparticle mass, i.e., $m_1/m' = m'/m_1$. In this case, we infer that critical angles exist only in a limited, *M*-dependent range of *Y* values and therein they admit a minimum. We point out that even though it is difficult to measure the transmission angle in the S side, it is nonetheless possible to control the current injection from the F side¹⁸ and consequently estimate the critical angle for current transmission. It is worth pointing out that a measurement of the transmission critical angle may be used to estimate the mass asymmetry, if the value of the magnetization of the F side is known by some other way.

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IV. CONCLUSIONS

We have investigated the conductance spectra and the critical angles of an F/I/S junction for different types of itinerant ferromagnets using the BTK method and solving the corresponding BdG equations. A special emphasis has been devoted to the role played by the exchange splitting and spin-dependent mass asymmetry in the F side. Namely, the study performed here unequivocally shows that the junction can be considered an efficient device to probe the itinerant ferromagnetism. Moreover, a measurement of the transmission critical angle allows for the determination of the possible mass asymmetry among spin-up and spin-down carriers. Thus, the F/I/S junction turns out to be a valuable device to probe ferromagnetism by providing also the possibility to estimate the mass mismatch of carriers. It is worth stressing that such kind of system may be easily realized using wellconsolidated fabrication procedures and standard measurements can be performed on it. As shown previously, clear-cut results may be obtained carrying out the experimental investigation in the metallic as well as in the tunneling regime.

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